Theoretical Description of Electron Cooling of HCI in the HITRAP Cooler Trap[1]

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- Subjects, questions:
  - Cooling times
  - Recombination losses
- Ultimate aim: evolution of the phase-space distribution $f(\vec{R}, \vec{V}, t)$ of the ions
- Description of the cooling process comprises:
  - Energy loss of ions in magnetized electrons
  - Heating of electrons by the ions
  - Cooling of electrons by synchrotron radiation

Operational parameters:

- $Z = -1, 1 \dots 92, \ A/Z \leq 3$
- $B \approx 6$ T
- $T_{e,0} \approx 4$ K
- $n_e \approx 10^7 \dots 10^8$ cm^{-3}
- $N_e \approx 10^8 \dots 10^{10}$
- $N_i \approx 10^5$
- $N_i/N_e \approx 10^{-5} \dots 10^{-3}$

[1] supported by BMBF and GSI

The cooling processes: basic equations and assumptions*

- Energy loss of ions and transfer of the released energy to the trapped electrons

$$\sum_{\mu}^{N_i} \frac{dE_\mu}{dt} = \sum_{\mu}^{N_i} M \frac{d\vec{V}_\mu}{dt} \cdot \vec{V}_\mu = -\frac{dE_e}{dt} \stackrel{!}{=} -\frac{3}{2} N_e k_B \frac{dT_e}{dt}$$

- Heating of the electrons by the ions and cooling by synchrotron radiation ($\tau \approx 0.1s$) to $T_0 (= 4K)$

$$\frac{dT_e}{dt}(t) = -\frac{2}{3 k_B N_e} \sum_{\mu}^{N_i} \frac{dE_\mu}{dt}(t) \; - \; \frac{1}{\tau} (T_e - T_0)$$

- Assumptions/Simplifications
  - Ion energy instantaneously converted into the electron temperature T_e
  - Isotropic e^- distribtuion

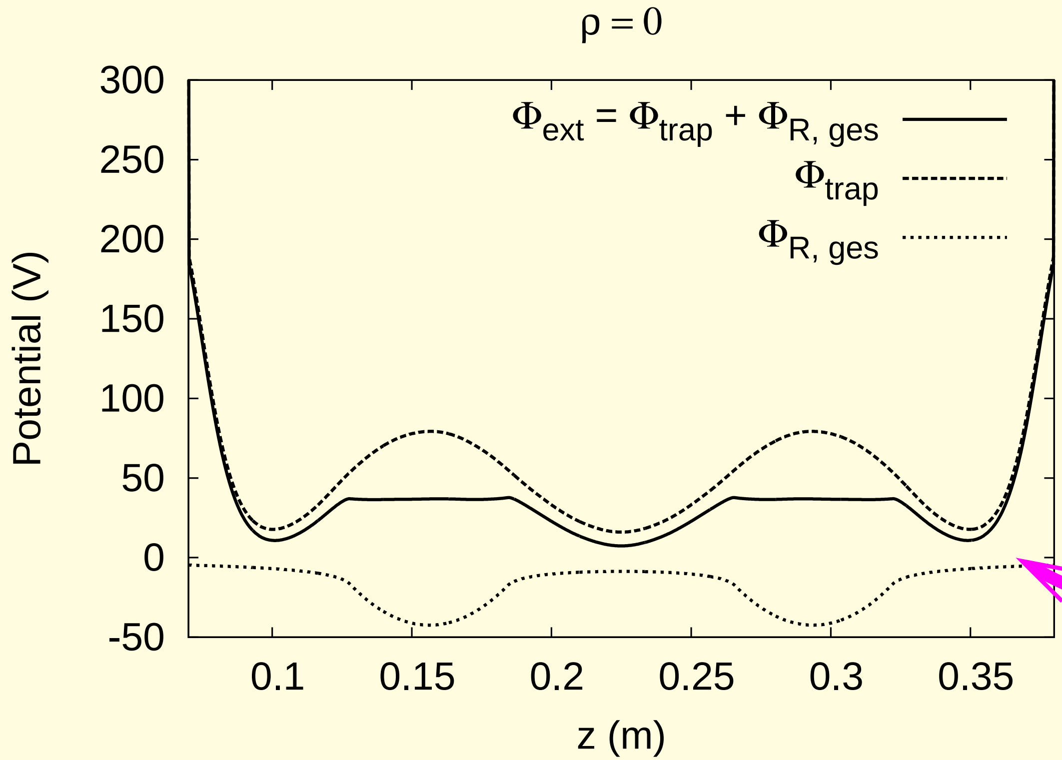
* See also: S.L. Rolston, G. Gabrielse, Hyp.Int. **44**, 233 (1988); J. Bernard et al., NIMA **532**, 224 (2004).

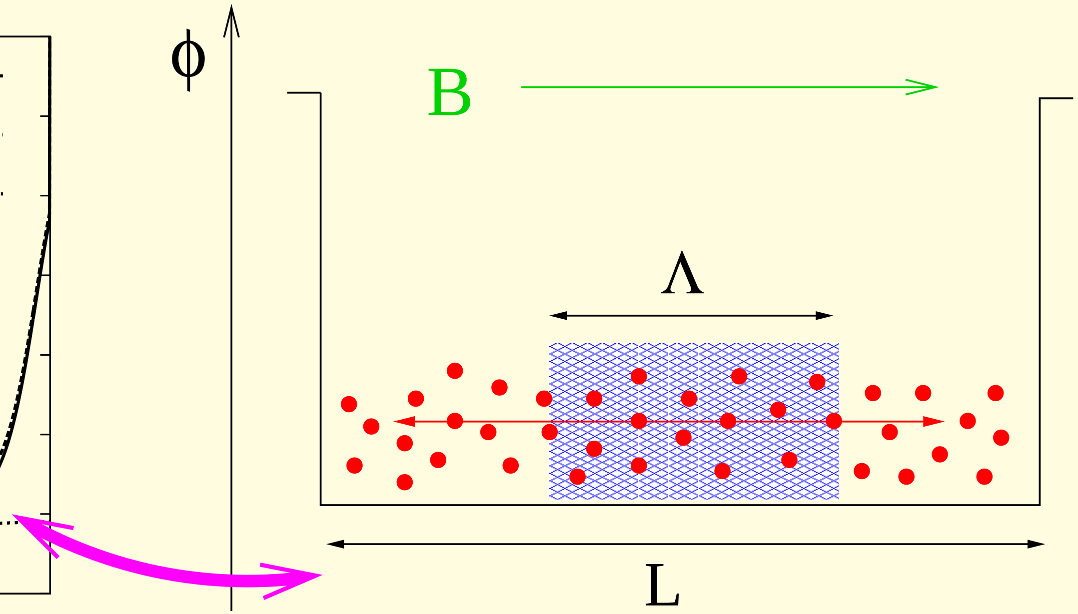
- Motion of ions and deceleration by collisions with magnetized electrons:

$$M\frac{d\vec{V}_\mu}{dt} = \vec{F}[n_e, T_e(t), \vec{B}, \vec{V}_\mu] + \, Ze\left(-\vec{\nabla}\Phi(\vec{R}_\mu) \, + \, \vec{V}_\mu \times \vec{B}\right), \qquad \frac{d\vec{R}_\mu}{dt} = \vec{V}_\mu$$

with $\vec{F}$ from microscopic calculations and simulations of the stopping force on ions in strongly magnetized electrons

- Approximations made for $\Phi(\vec{r})$ in the present studies: No ion-ion interaction, HITRAP design* $\longleftrightarrow$ Square well trap potential, cylindrical electron cloud





$\Lambda/L \approx 0.25$

* G. Maero, private communications

Recombination

- Calculation of the radiative recombination rate $\nu_{RR}(t)$ **for each ion**:
  - Ion-electron RR-cross section (e.g. M. Pajeck, R. Schuch, NIMB **93**, 241 (1994).):

$$\sigma_{\mathrm{RR}} = \sigma_0 \left(\frac{0.161}{\tilde{v}_r^2} - \frac{\ln \tilde{v}_r}{\tilde{v}_r^2} + \frac{0.518}{\tilde{v}_r^{4/3}} + \frac{0.074}{\tilde{v}_r^{2/3}} + 0.046 \ln \tilde{v}_r + 0.068 \right)$$

with $\sigma_0 = 2.1 \cdot 10^{-22}\ \mathrm{cm}^2$, $\tilde{v}_r^2 = \dfrac{m_e v_r^2}{2Z^2 13.6\mathrm{eV}}$ and $\vec{v}_r = \vec{V} - \vec{v}_e$

- Actual recombination rate using $V(t)$ and $T_e(t)$:

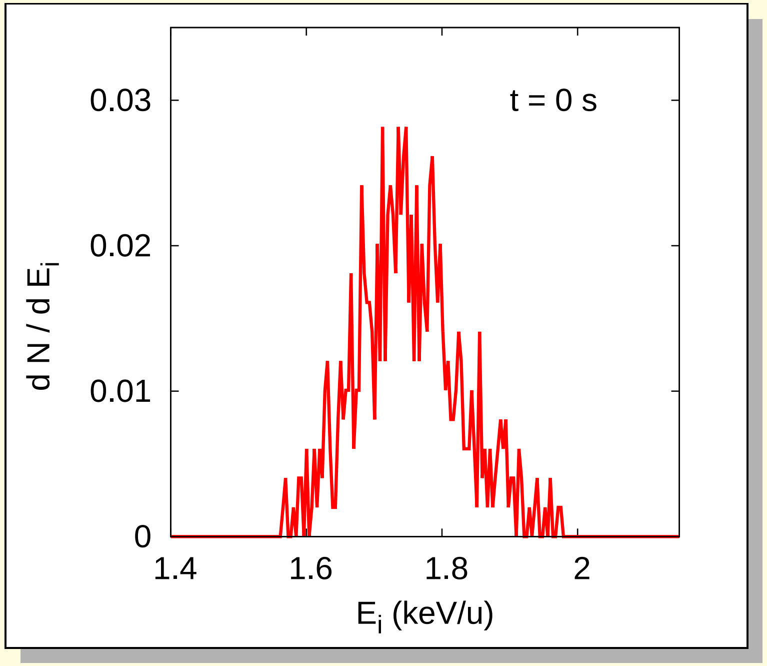
$$\nu_{\mathrm{RR}}(t) = n_e \int d^3 v_e\ v_r(t)\ \sigma_{\mathrm{RR}}(v_r(t))\ \left(\frac{m_e}{2\pi k_B T_e(t)} \right)^{3/2} \exp\left(-\frac{m_e v_e^2}{2 k_B T_e(t)} \right)$$

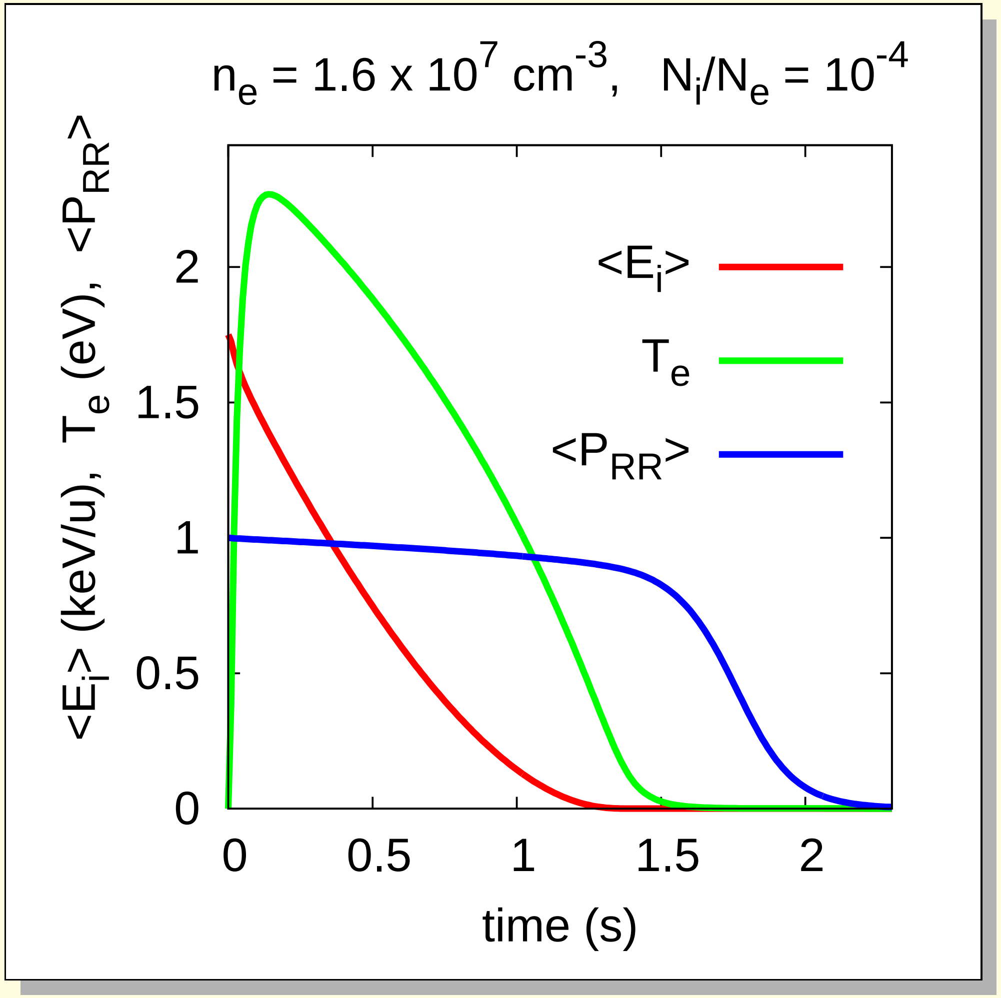
- Surviving probability (probability for remaining in the initial charge state):

$$P_{\mathrm{RR}}(t) = \exp\left(- \int_0^t dt' \nu_{\mathrm{RR}}(t') \right)$$

Cooling of U^{92+} and heating of electrons in HITRAP

- U^{92+}, $T_e(0) = 4$ K, $B = 6$ T



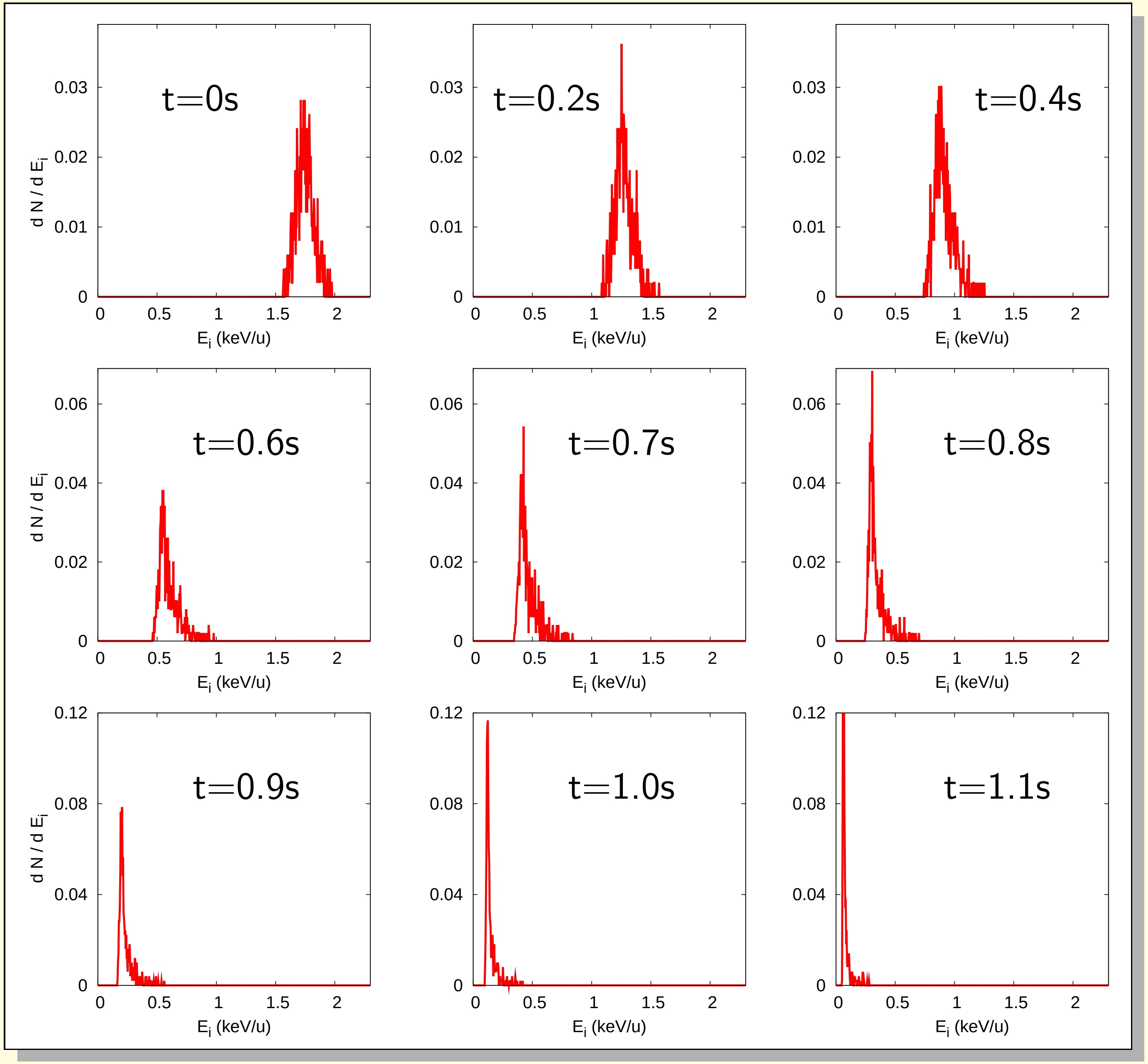


- Initial ion distribution with $N_i = 500$ ions (representing 10^5 trapped ions) as obtained from simulations of the injection into the HITRAP cooler trap*

∗ F. Herfurth, private communications

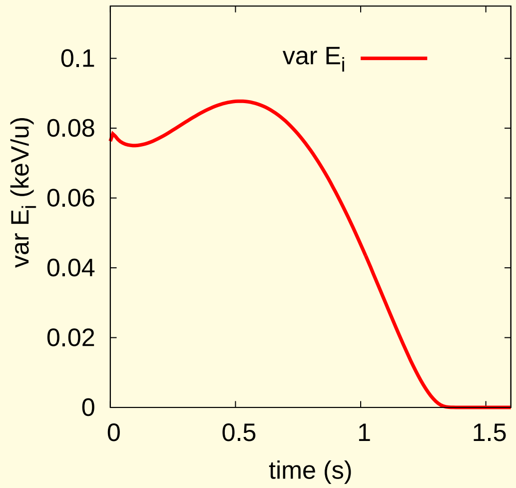
■ Time evolution of the ionic energy distribution ($n_e = 1.6 \times 10^7 \text{cm}^{-3},\ N_i/N_e = 10^{-4}$)

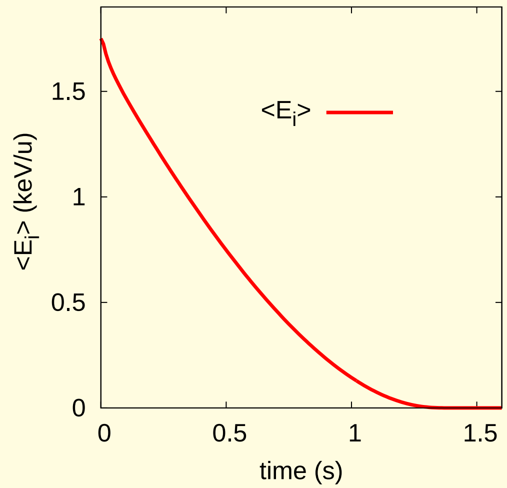
$\frac{dN}{dE_i}$



E_i (keV/u)

width $(E_i^2 - <E_i>^2)^{1/2}$

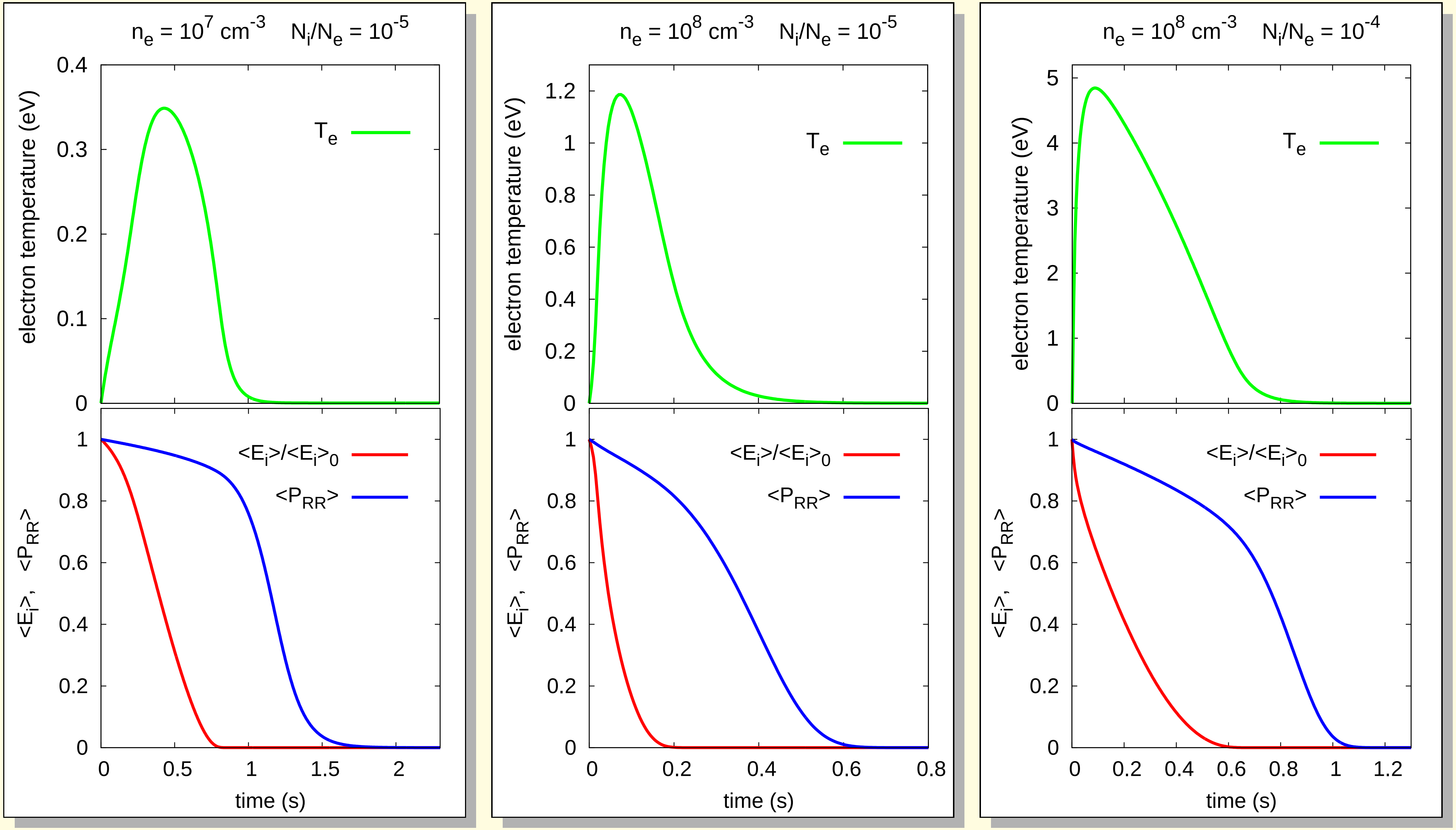




$< E_i >$

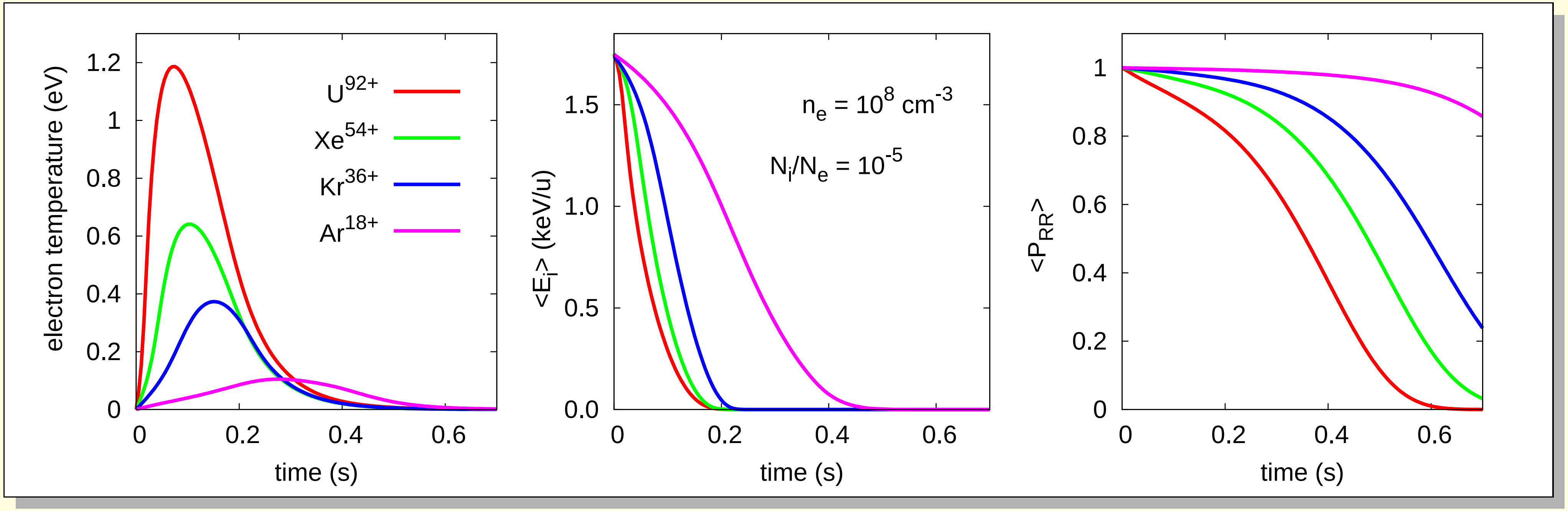
■ U^{92+}: Electron temperature T_e, Ion energy $\langle E_i \rangle$, Surviving probability $\langle P_{\mathrm{RR}} \rangle$

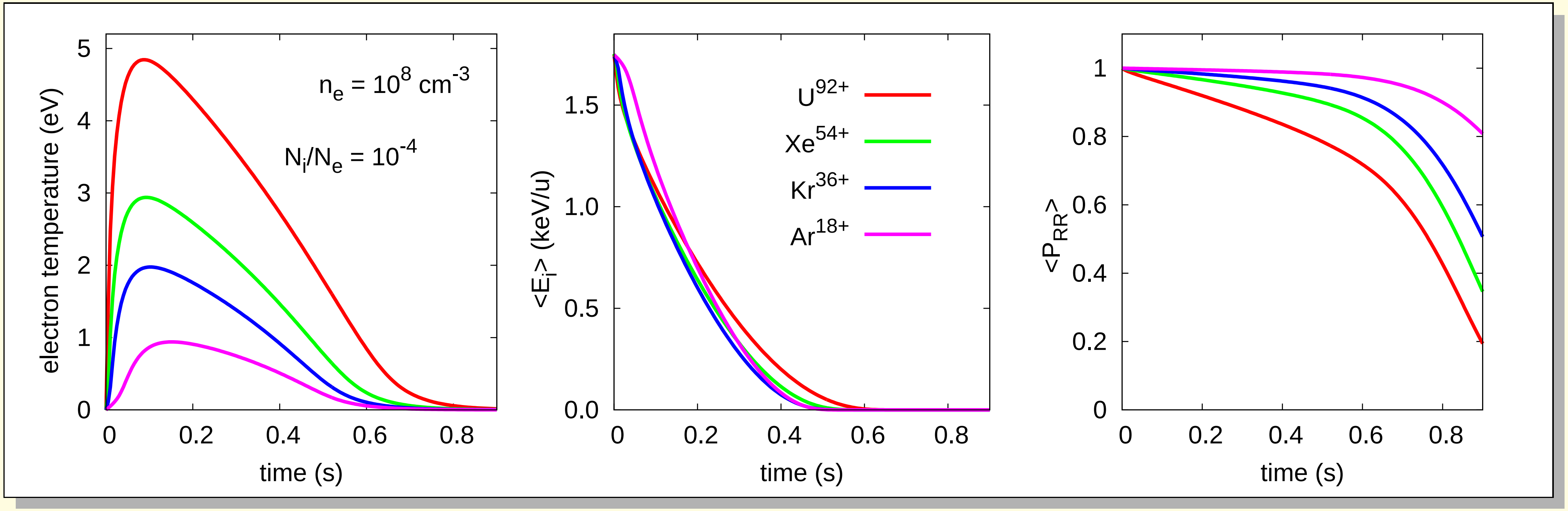
- Some examples for varying ratios of ions to electrons N_i/N_e and electron densities n_e



◀▶ Intricate feedback between $\frac{dE_i}{dt}$ and $\frac{dT_e}{dt}$ due to the nonlinear dependency of $\vec{F}$ on T_e

■ Electron temperature, Ion energy and Surviving probability for different HCI





Summary and Outlook

- Cooling times and recombination losses depend nonlinearly on N_i/N_e and n_e due to the feedback between electron heating and cooling forces

- Cooling times < 1 s and RR-losses $< 10\%$ feasible for HCI in HITRAP at $n_e \approx 10^7 \dots 10^8$ cm^{-3}, $N_e \approx 10^9 \dots 10^{10}$, and $N_i \approx 10^5$

- Open tasks and questions

  - Anisotropic energy transfer $\leftrightarrow$ anisotropic e^--distribution $\leftrightarrow$ $T_{e,\perp}, T_{e,\parallel}$?
    - Cooling force: $\vec{F}[T_e] \rightarrow \vec{F}[T_{e,\perp}, T_{e,\parallel}]$
    - Coupled rate equations for $T_{e,\perp}$ and $T_{e,\parallel}$
  - Selfconsistent space charge fields and ion-ion collisions
  - Instabilities?

Anisotropic energy transfer?

- Ion energy is transfered anisotropic (at strong B):

$$\sum_{\mu}^{N_i} \frac{dE_\mu}{dt} = -\frac{dE_{e,\parallel}}{dt} = -\frac{k_B N_e}{2} \frac{dT_{e,\parallel}}{dt}$$

- Radiative cooling of perpendicular components only:

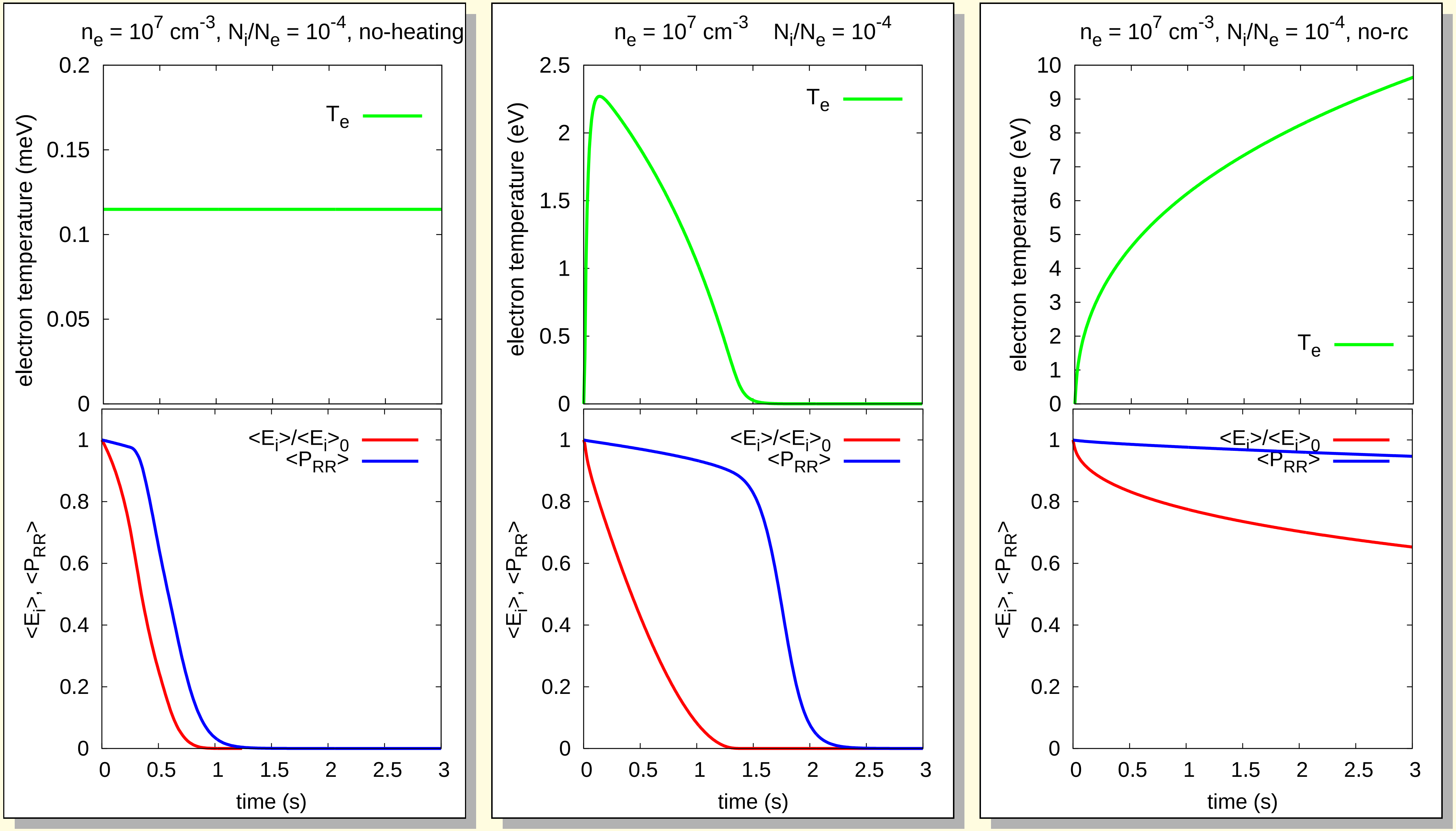
$$\frac{dT_{e,\perp}}{dt} = -\frac{1}{\tau}(T_e - T_0)$$

- Requires treatment of heating, radiative cooling and isotropization?

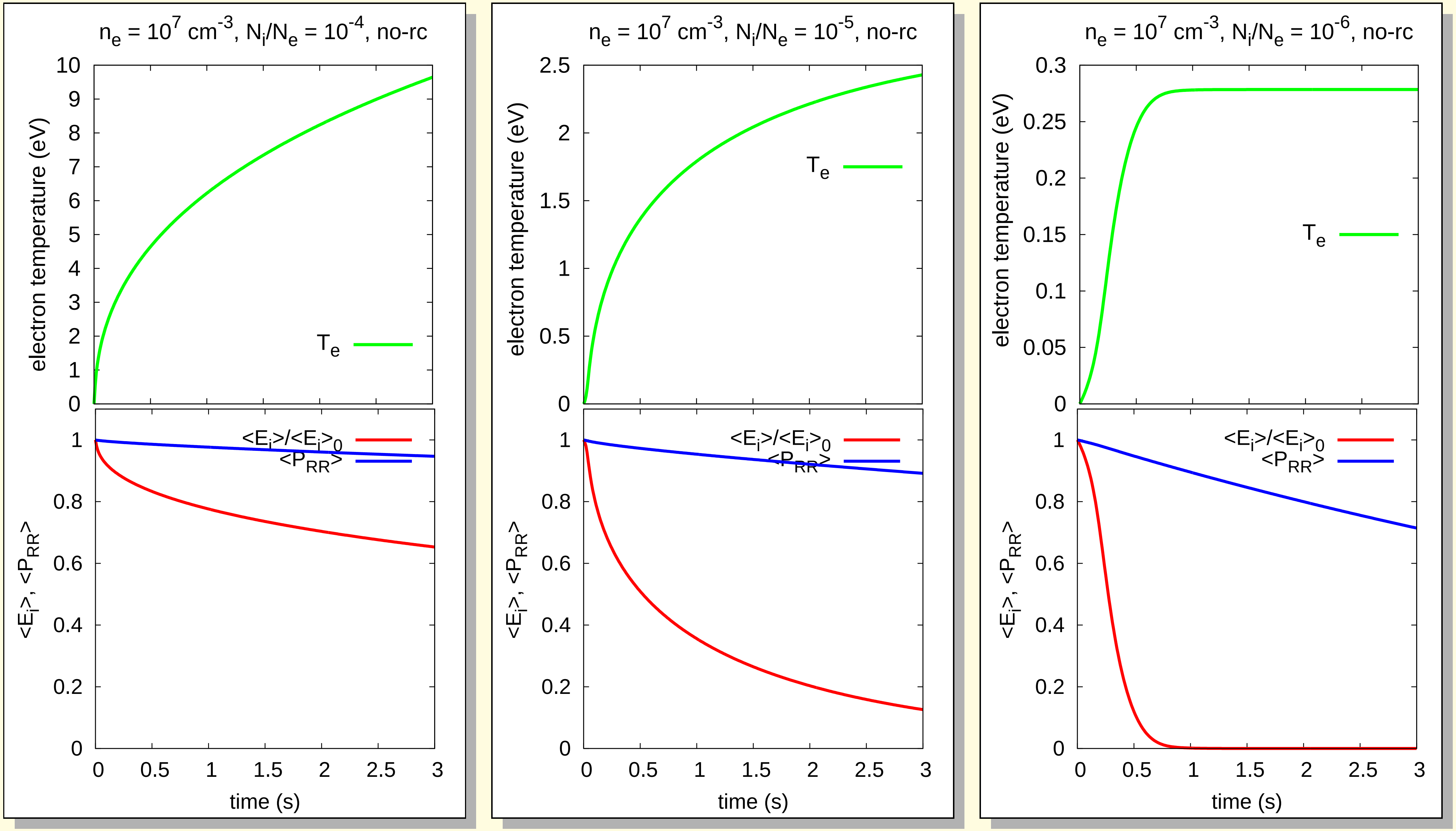
$$\frac{dT_{e,\parallel}}{dt} = -\frac{2}{N_e k_B} \sum_{\mu}^{N_i} \frac{dE_\mu}{dt} - 2\nu_{iso}\left(T_{e,\parallel} - T_{e,\perp}\right)$$

$$\frac{dT_{e,\perp}}{dt} = \nu_{iso}\left(T_{e,\parallel} - T_{e,\perp}\right) - \frac{1}{\tau}(T_e - T_0) \qquad \text{with} \qquad \nu_{iso} \ll \frac{1}{\tau} \; !?$$

U^{92+}: Electron temperature T_e, Ion energy $\langle E_i \rangle$, Surviving probability $\langle P_{\mathrm{RR}} \rangle$



■ U^{92+}: Electron temperature T_e, Ion energy $\langle E_i \rangle$, Surviving probability $\langle P_{RR} \rangle$



U^{92+}: Electron temperature T_e, Ion energy $\langle E_i \rangle$, Surviving probability $\langle P_{\text{RR}} \rangle$