Towards a *g*-factor determination of the electron bound in highly-charged calcium ions

Hitrap Workshop GSI November 2006

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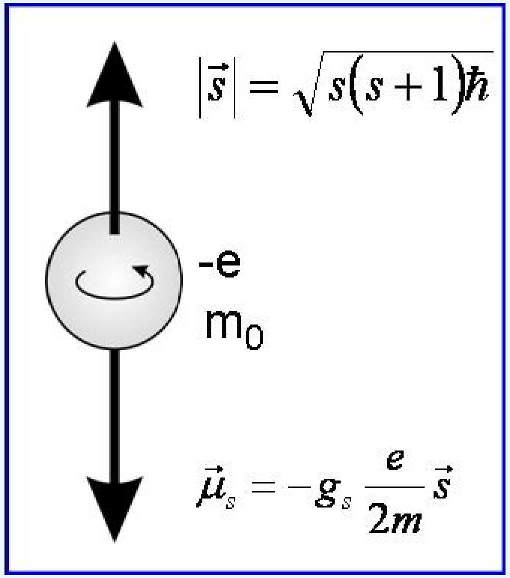
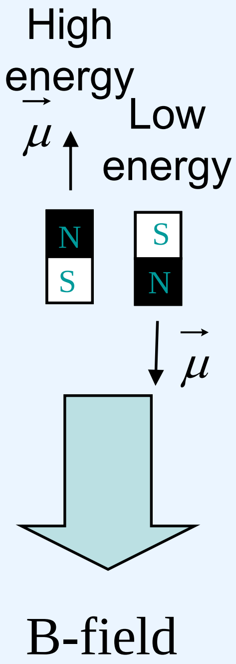
What is the *g*-factor?
- **Motivation**
- **How to measure the *g*-factor**

g-factor





- **A particle with a magnetic moment ❍ experiences a torque in a magnetic field B**

$$\Delta E = -\vec{\mu}\cdot\vec{B}$$

- **Both the orbital and the spin angular momentum contribute to the magnetic moment:**

$$\vec{\mu}_l = -\frac{e}{2m}\vec{L}$$

$$\vec{\mu}_s = -g\frac{e}{2m}\vec{S}$$

The *g*-factor throughout history

Classical Electrodynamics
g-factor does not exist

O. Stern and W. Gerlach (1922)

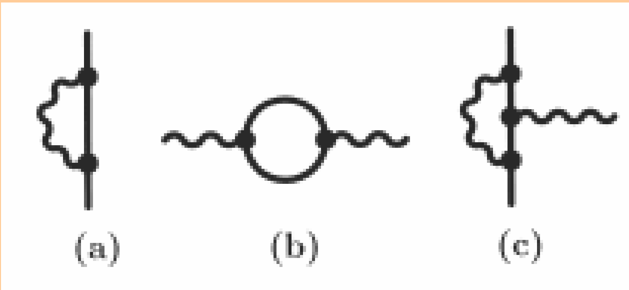
Quantum Mechanics
Dirac's equation predicts g = 2

Relativistic Quantum Mechanics
Breit (1928)

$$g = 2\left[\frac{1+2\sqrt{1-(Z\alpha)^2}}{3}\right]$$

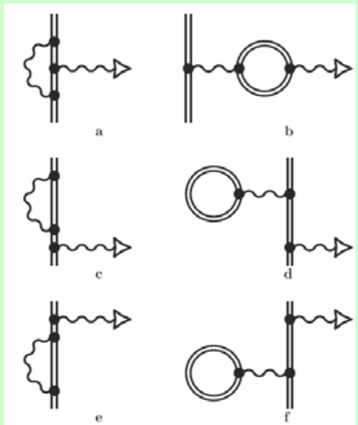
for point-like nucleus

Quantum Electrodynamics
Schwinger (1947)



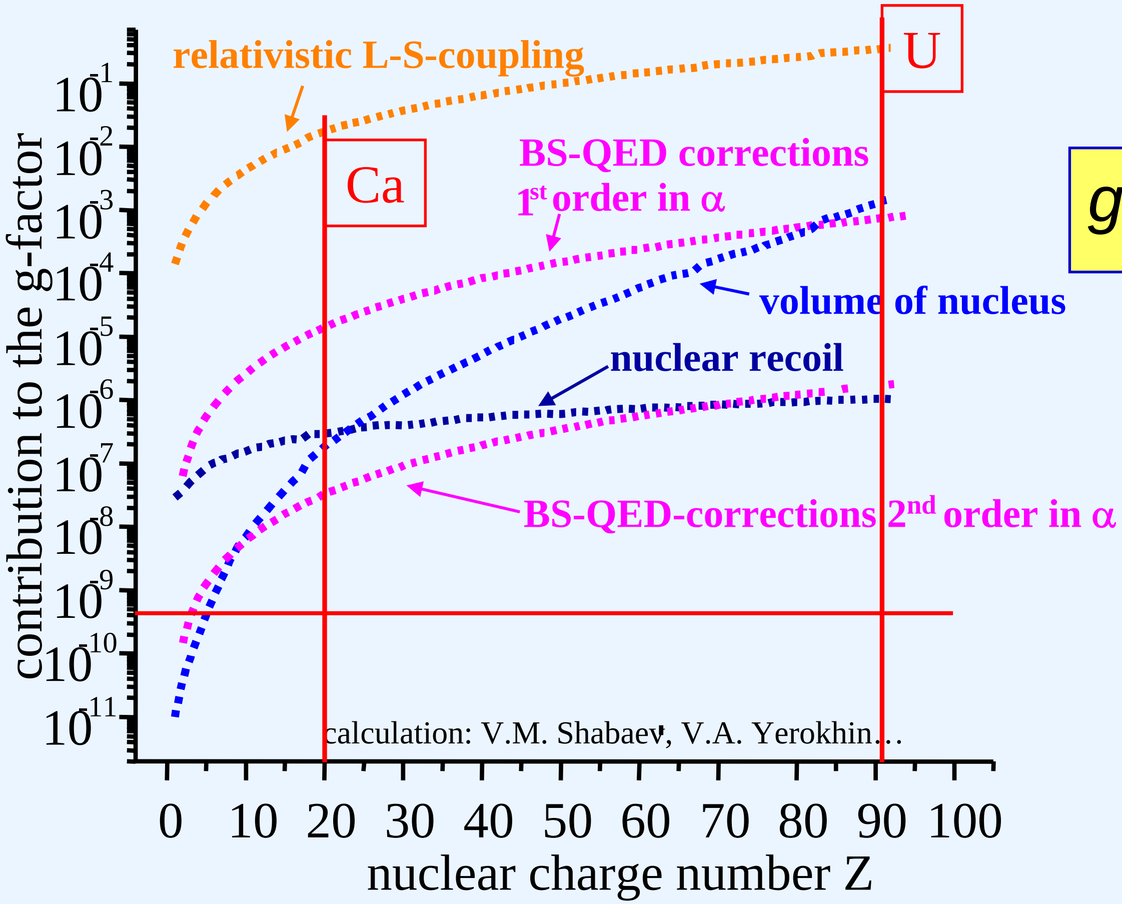
Self energy — Vacuum polarization — Vertex correction

Bound state – QED
Interaction with e.m. field from nucleus



Recoil and nuclear corrections
Mass, size and shape of the nucleus
Beier (2000) and Pachucki (2005)

Bound - state QED



Dirac — BS-QED — Nuclear correction

$$g = 2 + a_e + a_b + b + c$$

QED free electron — Relativistic

Contribution	$^{40}Ca^{19+}$
Dirac value (point)	1.9857232037 (1)
QED total	+0.002333332
Finite-size correction	+0.0000001130 (1)
Recoil	+0.0000002973
Interelectronic interaction	-
Theory total	1.9880569466 (100)
Bound-state QED only	0.000014414

[Pachucki et al, PR (2005)]

Why do we want to use Ca ions?

strong electric field ~ 10^{14} V/cm

Achievements

$^{12}C^{5+}$

g_{expt}=2.001 041 596 3 (10)(44) H. Häffner et al. PRL 85 (2000) 5308

g_{theo}=2.001 041 590 18 (3) K. Pachucki et al. PRA 72 (2005) 022108

$^{16}O^{7+}$

g_{expt}=2.001 047 026 0 (15)(44) J. Verdú et al. PRL 92 (2004) 093002

g_{theo}=2.001 047 020 32 (11) K. Pachucki et al. PRA 72 (2005) 022108

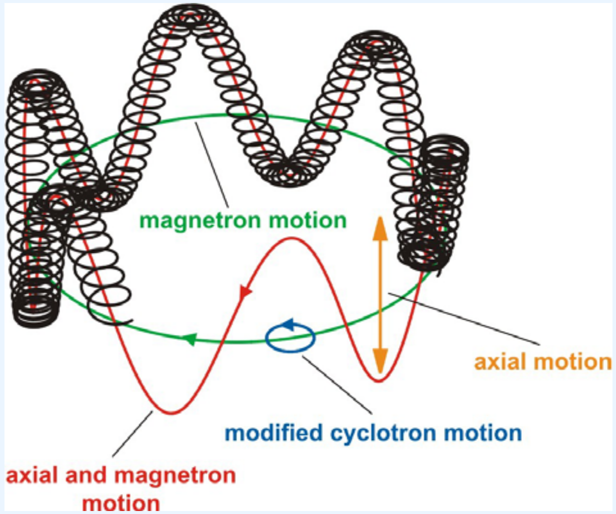
Resulting electron mass

m=0.000 548 579 909 2 (4) u T. Beier et al, PRL 88 (2002) 011603

(which is the main contribution to the currently accepted CODATA value)

Next step: $^{40}Ca^{17+}$, $^{40}Ca^{19+}$, $^{48}Ca^{17+}$, $^{48}Ca^{19+}$

Single ion in a Penning trap



reduced cyclotron frequency: $\omega_+ = \frac{\omega_c}{2} + \sqrt{\left(\frac{\omega_c}{2}\right)^2 - \frac{\omega_z^2}{2}}$

magnetron drift frequency: $\omega_- = \frac{\omega_c}{2} - \sqrt{\left(\frac{\omega_c}{2}\right)^2 - \frac{\omega_z^2}{2}}$

axial oscillation frequency: $\omega_z = \sqrt{\frac{qU_0}{Md^2}}$

ω_+ = 25 MHz | **ω_- = 16 kHz** | **ω_z = 1 MHz**

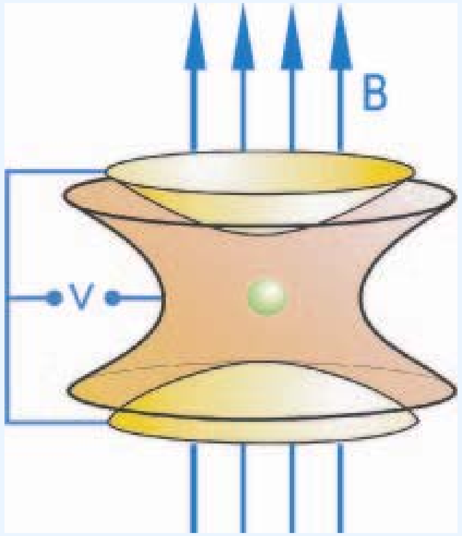
($^{40}Ca^{19+}$ @ B = 3.8 T; U_0 = 10 V)

invariance theorem:

$$\omega_c^2 = \omega_+^2 + \omega_-^2 + \omega_z^2$$

cyclotron frequency:

$$\omega_c = \frac{q}{M} B$$



g-factor measurement

Cyclotron frequency:

$$\omega_c = \frac{q}{M} B$$

Larmor frequency:

$$\omega_L = g \frac{e}{2m_e} B$$

$$g = 2 \frac{\omega_L}{\omega_c} \frac{q}{e} \frac{m_e}{M}$$

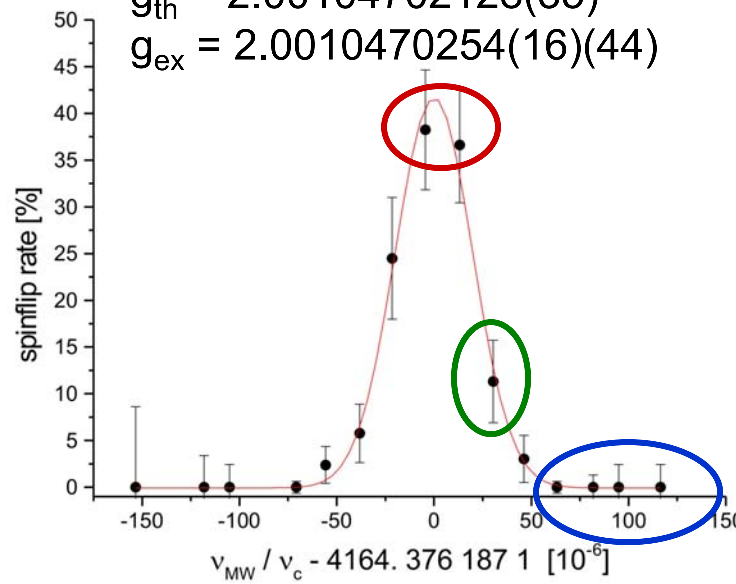
cyclotron frequency:

from literature

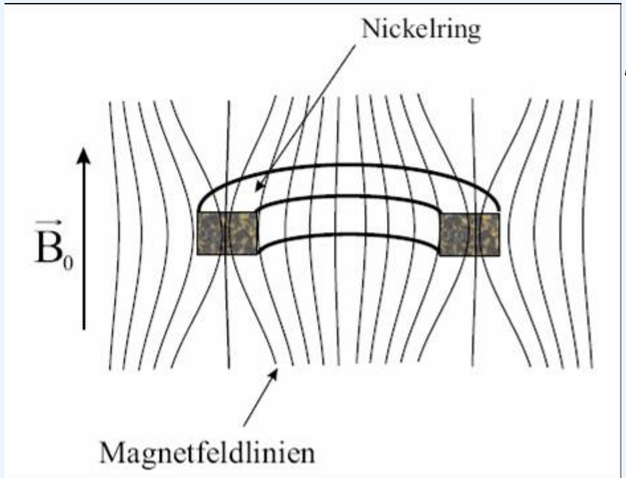
m(^{40}Ca): [SMILETRAP: Nagy et al, EPJD (2006)]

g_{th} = 2.00104702128(35)

g_{ex} = 2.0010470254(16)(44)



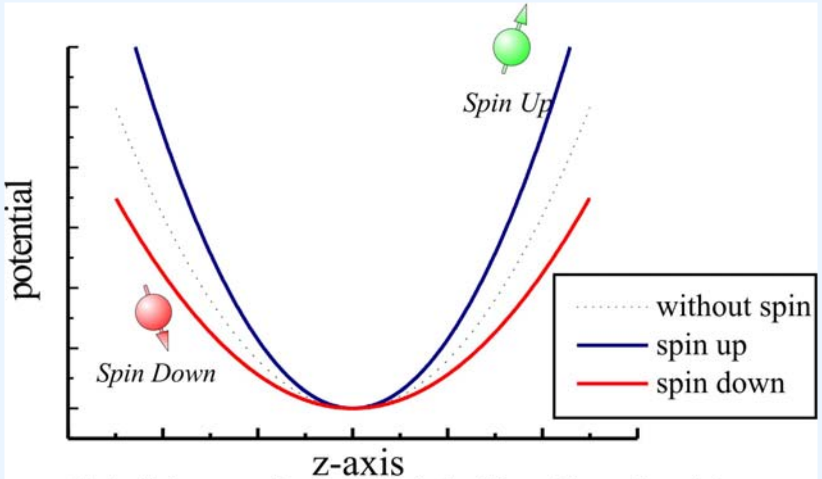
[Verdú et al, PRL 92 (2004) 093002]



***B*$_2$ = 10.06 (6) mT/mm^2**

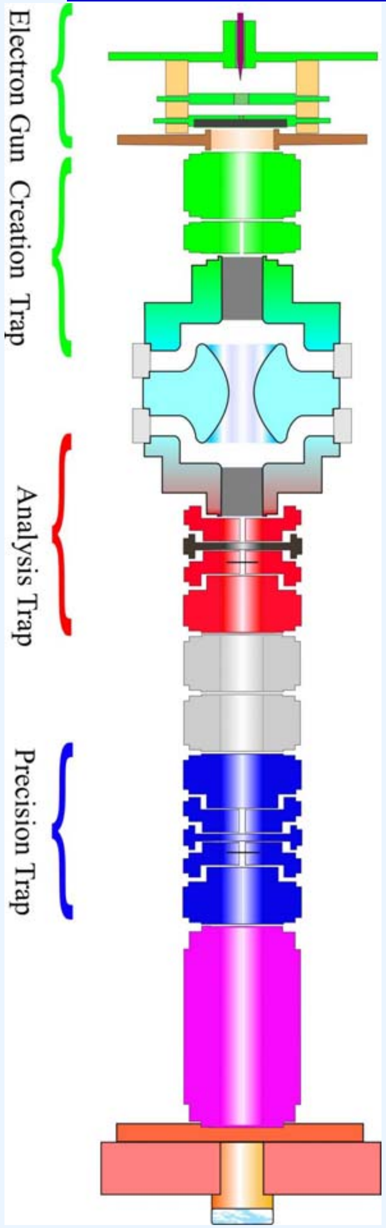
$$\delta\omega_z = 2\frac{g\mu_B B_2}{M\omega_z}$$

$\Delta\omega_z$ = 180 mHz between spin ↑↓



Potential as seen by the particle in different hyperfine states.

The calcium triple Penning trap



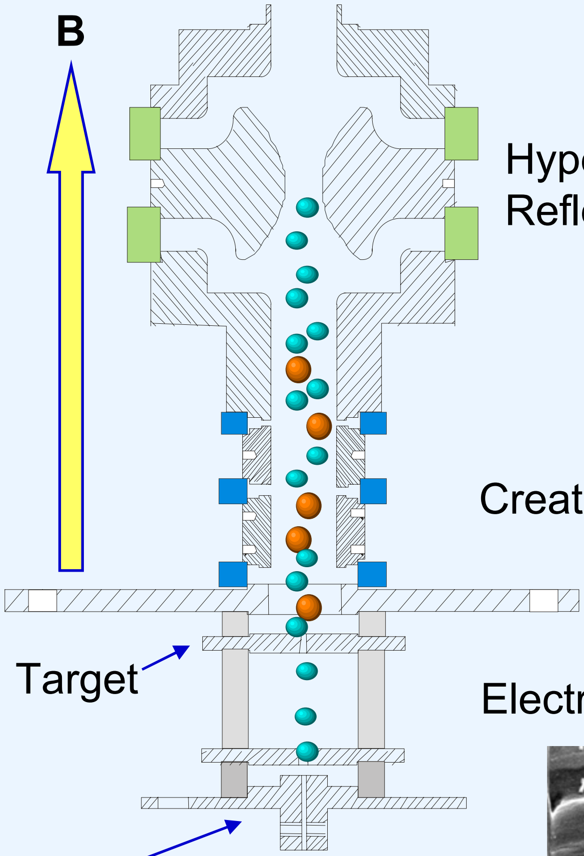
- **Electron gun and creation trap:**
  - charge breeding
- **Analysis trap:**
  - inhomogeneous magnetic field (B_2 = 8,2(9) mT/mm^2)
  - monitor spin direction
- **Precision trap:**
  - homogeneous magnetic field
  - high precision measurements of ω_+, ω_-, ω_z of a single Ca ion + microwave irradiation (spinflip)

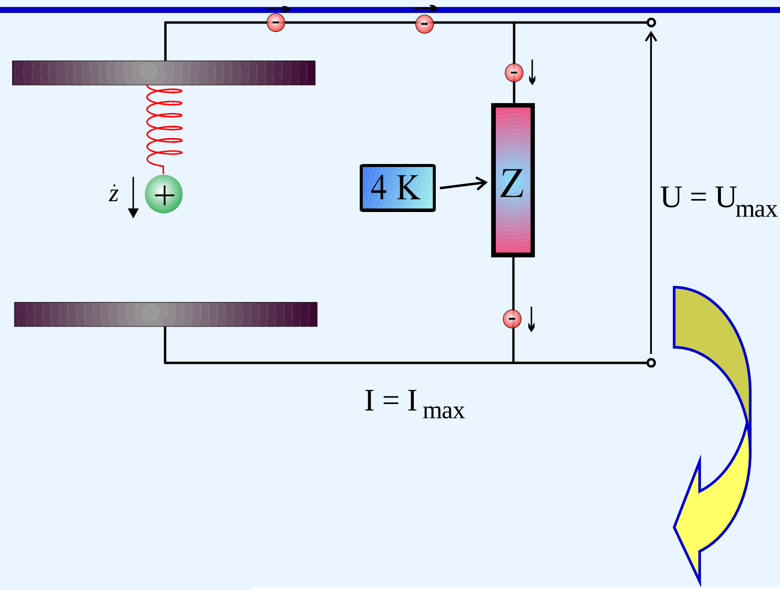
General:

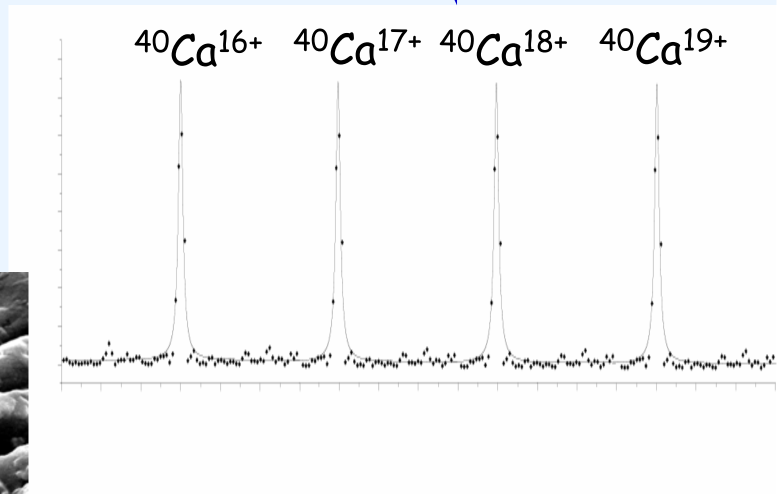
- temperature in the trap T = 4K
- pressure $p < 10^{-16}$ mbar

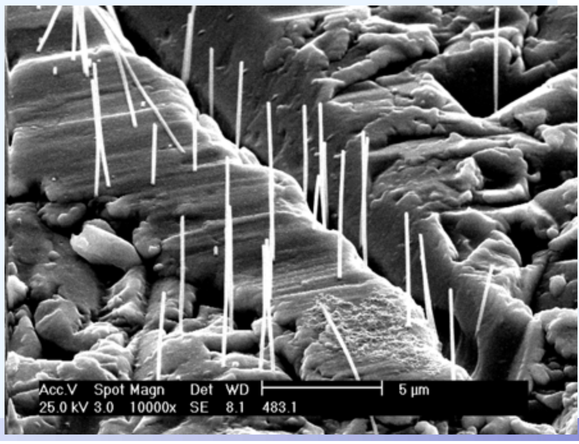
Charge breeding

"mini-EBIS"



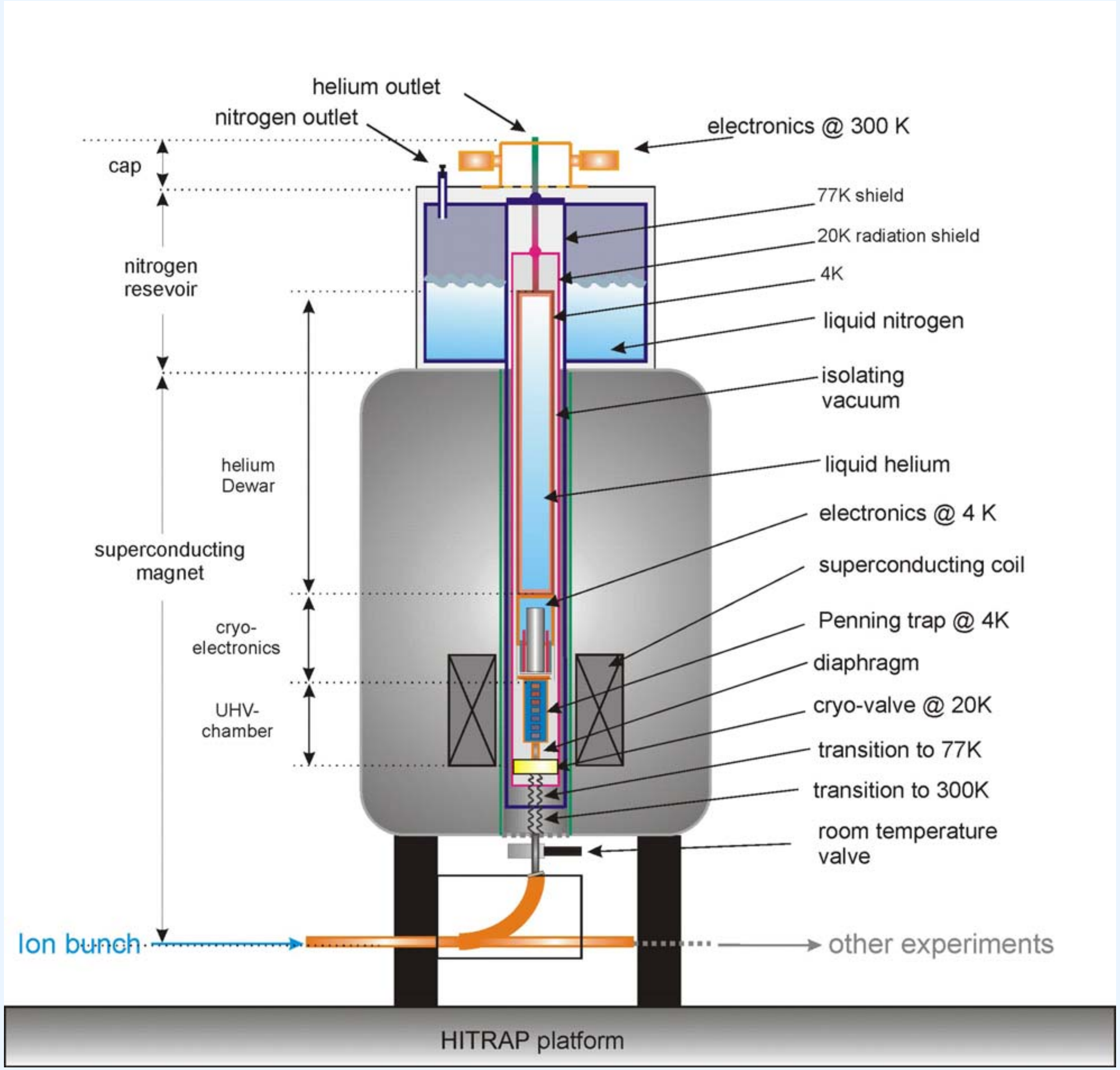






$$\omega_c = \frac{q}{m} B$$

g-factor of $^{238}U^{91+}$ @ HITRAP



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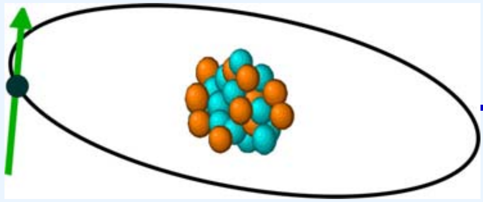
VH-NG-037

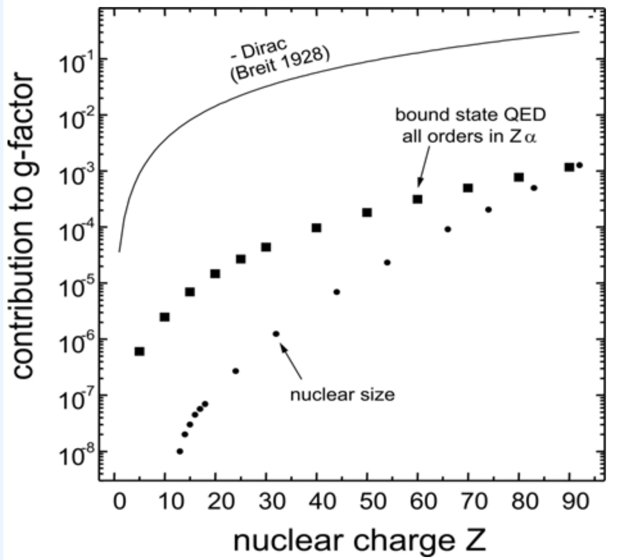
Thanks a lot for your attention!



Bound - state QED

Why do we want to use Ca ions?

strong electric field ~ 10^{14} V/cm



[Werth et al, Adv. At. Mol. Opt. Phys. 48 (2002) 191]

Dirac — BS-QED — Nuclear correction

$$g = 2 + a_e + a_b + b + c$$

QED free electron — Relativistic

for O^{7+}: [Verdú et al, PRL 92 (2004) 093002]

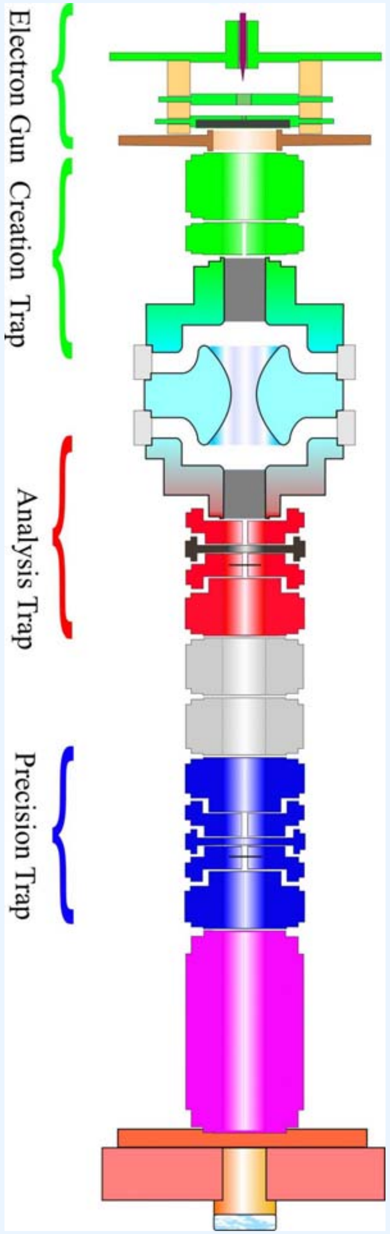
g_{th} = 2.00004702128(35)

g_{ex} = 2.0000470254(16)(44)

Contribution	$^{40}Ca^{19+}$	$^{40}Ca^{17+}$
Dirac value (point)	1.9857232037 (1)	1.996426011
QED total	+0.002333392	+0.002321708 (17)
Finite-size correction	+0.0000001130 (1)	+0.000000014
Recoil	+0.0000002973	+0.000000061 (2)
Interelectronic interaction	-	+0.00045445 (14)
Theory total	1.9880569466 (100)	1.99920224 (17)
Bound-state QED only	0.000014414	0.00000173

Measurement sequence

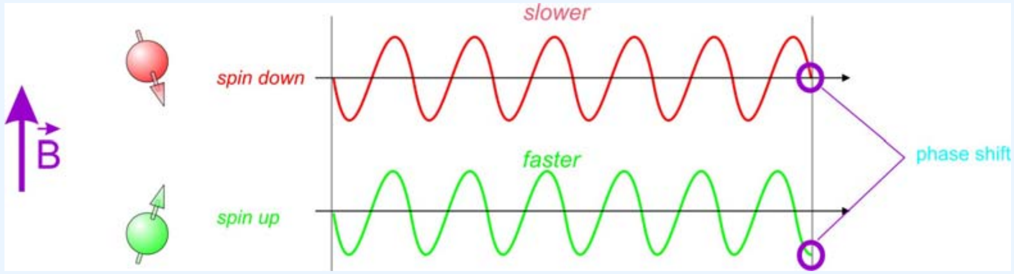
MATS

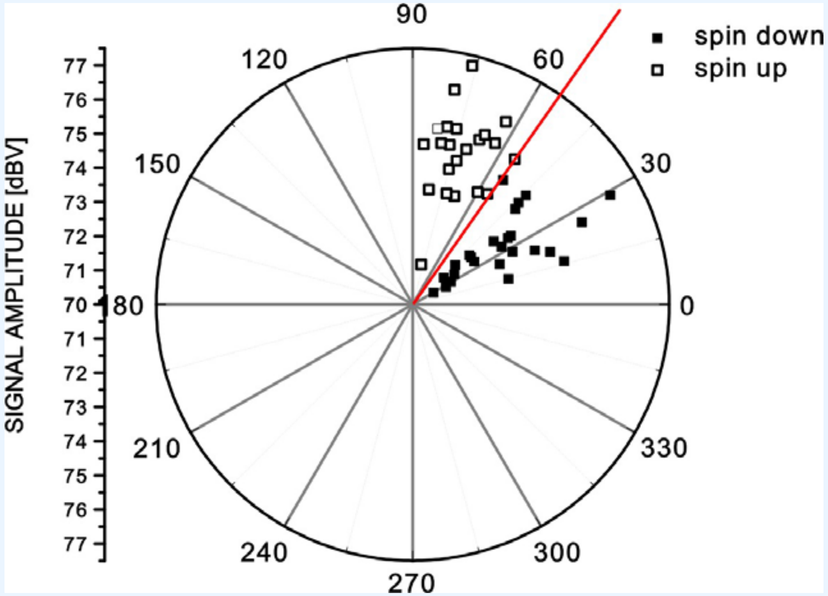


- **Creation trap:**
  - charged breeding until $^{40}Ca^{17+}$ or $^{40}Ca^{19+}$
- **Precision trap:**
  - clean the ion cloud until only one single Ca ion is left in the trap
  - prepare the spin direction of the single ion

- **Precision trap**
  - measurement of $\blacklozenge_{+}$ $\blacklozenge_{-}$ $\blacklozenge_{z}$ on a single ion with well-defined spin direction
    + microwave irradiation
- **Analysis trap:**
  - detection of spinflip
- **Precision trap:**
  start a new sequence…

Detection of spin direction – Phase-sensitive method





- Magnetic field inhomogeneity of the analysis trap:
  B_2 = 8,2(9) mT/mm^2

  Δ■$_z$ 🕓 180 mHz between spin ↑ ↓

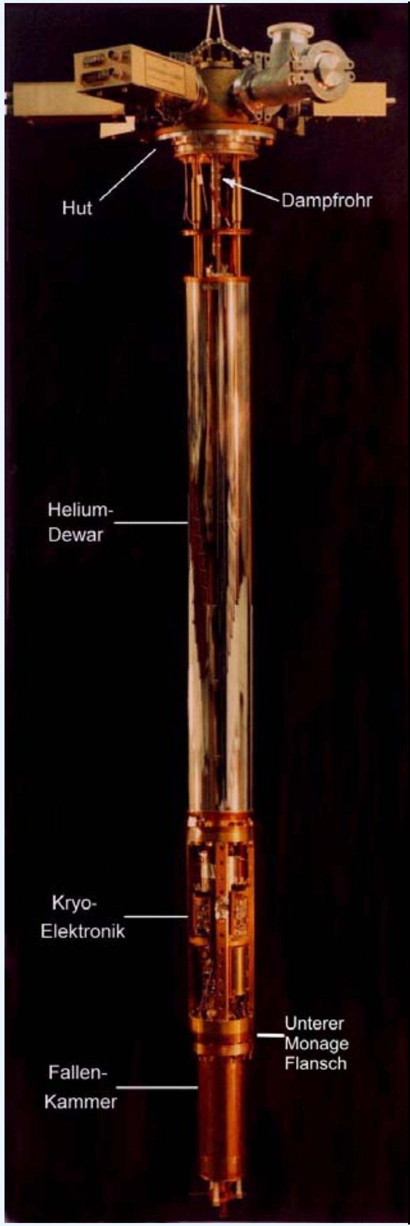
  - reducing the size of the analysis trap
  - materials with higher magnetic susceptibilities
  - higher stability by voltage supplies and detection techniques.

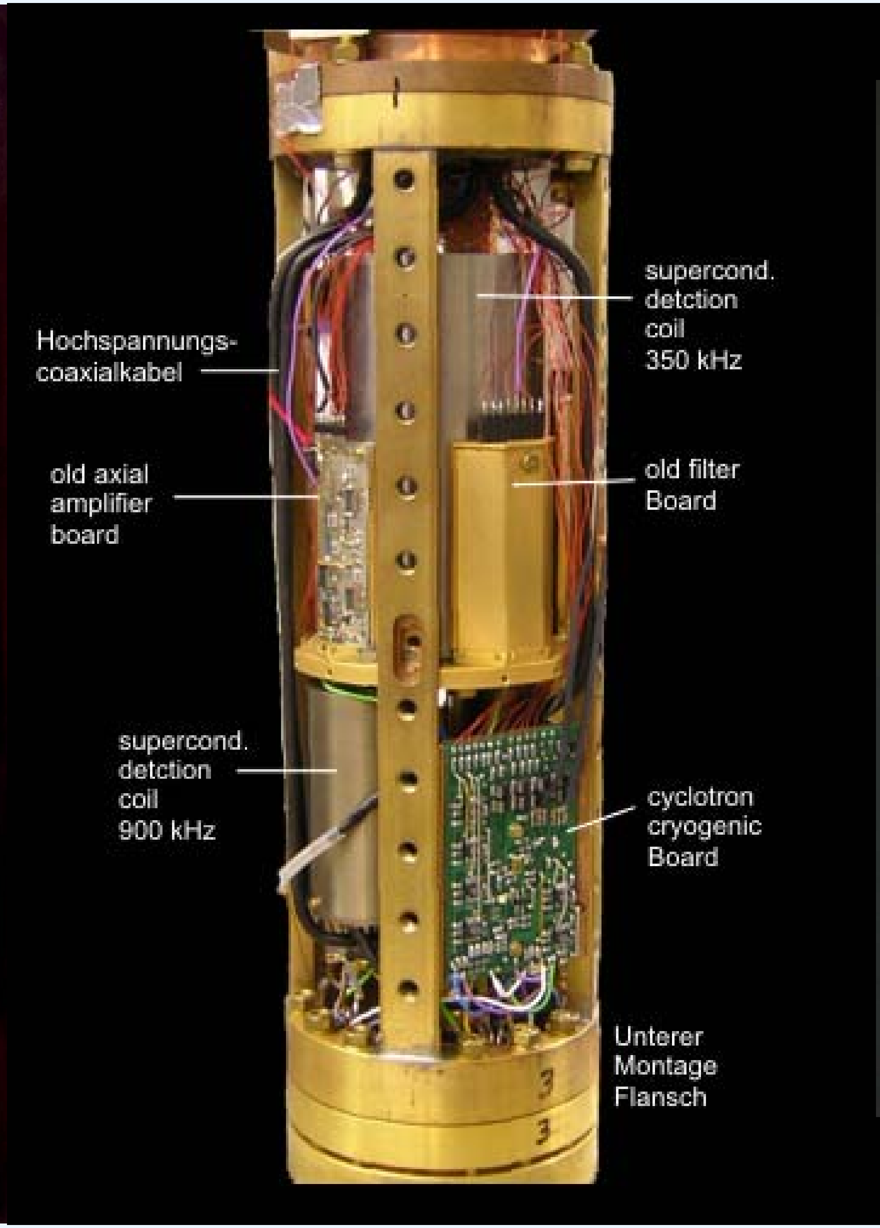
- Detection of spin flip:

  - measurement of the axial frequency
  - measurement of the phase
  - phase-sensitive method ~30 faster than measurement of ◆$_z$

No effect on the measurement of the frequencies in the precision trap
◆$_z$, ◆$_+$, ◆$_-$ ❾ ◆$_c$

Apparatus


Hut
Dampfrohr
Helium-
Dewar
Kryo-
Elektronik
Unterer
Monage
Flansch
Fallen-
Kammer


Hochspannungs-
coaxialkabel
supercond.
detction
coil
350 kHz
old axial
amplifier
board
old filter
Board
supercond.
detction
coil
900 kHz
cyclotron
cryogenic
Board
Unterer
Montage
Flansch

Cryo- Electronics

- ***FT-ICR board (inside the trap)***
- ***Old axial amplifier board***
- ***New axial amplifier board***
- ***Cyclotron cryogenic board***
- ***Old filter board***
- ***New filter board***