Reaction Parameters for Heavy-Ion Collisions

The relevant formulae are calculated if $\mathbf{A_1, Z_1}$ and $\mathbf{A_2, Z_2}$ are the mass (in amu) and charge number of the projectile and target nucleus, respectively, and **T** is the laboratory energy (in MeV).

Nuclear radius for homogeneous (sharp) mass distribution:

$$R_i = 1.28 \cdot A_1^{1/3} - 0.76 + 0.8 \cdot A_1^{-1/3} \quad [\,fm]$$

Nuclear radius for diffuse (Fermi) mass distribution:

$$C_i = R_i \cdot \left(1 - R_i^{-2}\right) \quad [\,fm]$$

Nuclear interaction radius:

$$R_{\mathrm{int}} = C_1 + C_2 + 4.49 - \frac{C_1 + C_2}{6.35} \quad [\,fm]$$

Reaction Parameters for Heavy-Ion Collisions

The relevant formulae are calculated if **A_1, Z_1** and **A_2, Z_2** are the mass (in amu) and charge number of the projectile and target nucleus, respectively, and **T** is the laboratory energy (in MeV).

For the grazing trajectory at the turning point R_{int} one obtains:

$$E_{cm} \cong T_\ell + V_{Coul} = \frac{\hbar^2}{2\mu}\frac{\ell_g \cdot (\ell_g + 1)}{R_{\text{int}}^2} + \frac{Z_1 \cdot Z_2 \cdot e^2}{R_{\text{int}}}$$

Grazing angular momentum ℓ_g:

$$\ell_g \cong k_\infty \cdot R_{\text{int}} \left(1 - \frac{2 \cdot a}{R_{\text{int}}}\right)^{1/2}$$

Impact parameter b_c for grazing collision:

$$b_c = a \cdot \cot\left(\frac{\theta_{1/4}^{cm}}{2}\right)$$

Distance of closest approach D (at turning point):

$$D = R_{\text{int}} = a \cdot \left[\sin\left(\frac{\theta_{1/4}^{cm}}{2}\right) + 1\right]$$

$$b_c = \sqrt{D^2 - 2 \cdot a \cdot D}$$

$$k_\infty = \frac{o.21872 \cdot A_2}{A_1 + A_2}\sqrt{A_1 \cdot T} \;\; [fm^{-1}] \qquad \eta = 0.15746 \cdot Z_1 \cdot Z_2 \cdot \sqrt{\frac{A_1}{T}} \qquad a = \frac{\eta}{k_\infty} = \frac{Z_1 \cdot Z_2 \cdot e^2}{p \cdot v_\infty} = \frac{0.72 \cdot Z_1 \cdot Z_2}{T} \cdot \frac{A_1 + A_2}{A_2} \;\; [fm]$$

Reaction Parameters for Heavy-Ion Collisions at Relativistic Energies

The relevant formulae are calculated if **A_1, Z_1** and **A_2, Z_2** are the mass (in amu) and charge number of the projectile and target nucleus, respectively, and **T** is the laboratory energy (in MeV).

Half-distance of closest approach a:

$$a = \frac{\eta}{k_\infty} = \frac{Z_1 \cdot Z_2 \cdot e^2}{p \cdot v_\infty}$$

With the relativistic momentum ($p = m_0 \cdot \gamma \cdot \beta \cdot c$ **):**

$$a = \frac{Z_1 \cdot Z_2 \cdot e^2}{m_0 \cdot c^2 \cdot \gamma \cdot \beta^2}$$

Impact parameter b_c for grazing collision:

$$b_c = a \cdot \cot\left(\frac{\theta_{1/4}^{cm}}{2}\right) \rightarrow 2 \cdot \tan\left(\frac{\theta_{1/4}^{cm}}{2}\right) = 2 \cdot \frac{a}{b_c}$$

Transformation of scattering angle:

$$\theta^{cm} \cong \vartheta_1^{\ell ab} + \arcsin\left(\frac{A_1}{A_2} \cdot \sin \vartheta_1^{\ell ab}\right) \rightarrow \vartheta_1^{\ell ab}$$

With $\cot \vartheta_1 = 1/\tan \vartheta_1$ **and** $\tan \vartheta_1 \cong \vartheta_1$ **one obtains for the angular deflectgion:**

$$\vartheta_1^{\ell ab} = \frac{2 \cdot Z_1 \cdot Z_2 \cdot e^2}{m_0 \cdot c^2 \cdot \gamma \cdot \beta^2 \cdot b} = \frac{2.88 \cdot Z_1 \cdot Z_2 \cdot [931.5 + (T/A_1)]}{A_1 \cdot [(T/A_1)^2 + 1863 \cdot (T/A_1)]} \cdot \frac{1}{b} \ [rad]$$

$$\beta = \frac{v_\infty}{c} = \frac{\sqrt{T^2 + 1863 \cdot A_1 \cdot T}}{931.5 \cdot A_1 + T} \qquad \gamma = \left(1 - \beta^2\right)^{-1/2} = \frac{931.5 \cdot A_1 + T}{931.5 \cdot A_1} \qquad \gamma \cdot \beta^2 = \frac{(T/A_1)^2 + 1863 \cdot (T/A_1)}{931.5 \cdot [931.5 + (T/A_1)]}$$